Chapter 7 Financial Context of Business III: Discounting and Investment appraisal

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Time value of money

Money received today is worth more than the same sum received in the future. This happens for several reasons:

- Potential for earning interest
- Impact of inflation
- Risk

The time value of money can be expressed in the form of annual interest rates. For investment appraisal this will be termed:

- Cost of capital
- Required return
- Discount rate

(a) Simple interest

- Interest is paid or received on the principal only

$$\boxed{\textbf{Interest} = P \times r \times n}$$

$$\boxed{\textbf{Future Value} = P + (P \times r \times n)}$$

P =amount invested

r = interest rate per annum as a decimal

n = number of years

Illustration of simple interest

- £1,000 is invested for 5 years. The sum earns 10% simple interest each year. How much will accumulate by the end of the fifth year?

Answer

- Future value at the end of year 5

= P + (P x r x n)

= £1,000 + (£1,000 x 0.1 x 5)

= £1,500

(b) Compound Interest

- Interest is paid or received on the principal plus any accumulated interest

$$\boxed{S = X(1 + r)^n}$$

- Formula is given
- S = value after n years
- X = amount invested
- r = annual rate of interest (as a decimal)
- n = number of years

Illustration of compound interest

- £1,000 is invested in an account for 5 years. The compound interest rate is 10% per annum. Find the value of the account (to the nearest pound) after 5 years and calculate the interest earned.

Answer

- Value after 5 years, $S = X(1 + r)^n$

$$= £1{,}000\,(1 + 0.1)^5$$

$$= \underline{£1{,}611}$$

- Interest = £1,611 - £1,000

$$= \underline{£611}$$

- The formula should be used in the exam but it may help to look at the calculation in this way:

Year 1: £1,000 + 10% interest = £1,000 x 1.1 = £1,100

Year 2: £1,100 + 10% interest = £1,100 x 1.1 = £1,210

Year 3: £1,210 + 10% interest = £1,210 x 1.1 = £1,331

Year 4: £1,331 + 10% interest = £1,331 x 1.1 = £1,464

Year 5: £1,464 + 10% interest = £1,464 x 1.1 = <u>£1,611</u>

(c) Interest paid more than once per annum

- Use the same formula for simple or compound interest but make sure the interest rate used matches the period of payment

Illustration of interest paid more than once per annum

- £500 is borrowed on a credit card charging 1.2% p.c.m. How much is owed after one year?

Answer

- One year is 12 months, so 12 periods of compound interest need to be applied.
- Balance after one year, $S = X(1 + r)^n$

  $= 500 \times (1.012)^{12} =$ <u>£577</u>, to the nearest £

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(d) Equivalent annual interest rates

- As mentioned, compound interest is often paid or received more than once a year
- It is useful to convert this period rate to an annual rate of interest
- This is called the equivalent annual interest rate or the annual percentage rate (APR)

Annual percentage rate = $(1 + r)^{n} - 1$

r = period interest rate (as a decimal)

n = the number of compounding periods in a year

Illustration of equivalent annual interest rates

- An account charges compound interest of 1% per month. Calculate the equivalent annual rate.

Answer

- APR = $(1.01)^{12} - 1 = 0.1268$ or <u>12.68%</u>
- Some financing companies can be economic with the truth when describing their products. The APR is usually the best indicator of the true cost.

Example 3

- £500 is invested in an account paying 1.25% interest per quarter.
  (a) Calculate the fund balance after 5 years.
  (b) Find the annual percentage rate.

(3) Sinking funds

- These have equal sums paid into them each period, e.g. a regular savings account
- Use the formula to calculate the amount at the end of the investment period

$$S = \frac{A\,(R^n - 1)}{(R - 1)}$$

S = amount at the end of investment period

A = Equal sum

R = 1 + interest rate (as a decimal)

n = number of periods

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Illustration of sinking funds (part 1)

- Suppose I pay £1000 a year into an account for 3 years at an interest rate of 10% with all payments made at the end of each year. How much will the fund accumulate to?

Answer

- Value after 3 years, $S = \frac{A(R^n - 1)}{(R - 1)}$

$$= \frac{£1{,}000\,(1.1^3 - 1)}{(1.1 - 1)}$$

$$= \underline{£3{,}310}$$

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Illustration of sinking funds (part 2)

- How would the answer differ if the funds were paid in at the start of each year?

Answer

- In the previous illustration we said that if the payments were made at the end of each year we would have £3,310 by the end of year 3
- However, if the payments are made at the start of each year they will attract an extra year’s interest and the final sum will be £3,310 x 1.1 = £3,641

(4) Discounting

- When we looked at compound interest we said that the future value after n periods, $S = X (1 + r)^n$
- However, we may know the future value, S, but need to calculate the present value, X. Rearranging the equation we get:

$$\boxed{\text{Present value, } X = \frac{S}{(1 + r)^n}}$$

S = future sum

n = number of periods

r = cost of capital/ discount rate as a decimal (we called this the interest rate previously)

Illustration of discounting (part 1)

- Find the present value of £25,000 receivable in 6 years' time, if the interest rate is 10% pa.

  (Calculate your answer to the nearest £)

Solution

- Present value, $X = \dfrac{S}{(1 + r)^n}$

$$= \frac{£25,000}{1.1^6}$$

$$= \underline{£14,112}$$

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Example 6

- Charlie wins a cash prize in a competition. He must choose between Prize A, B or C. The prize details are:

  Prize A: Receive £20,000 now.

  Prize B: Receive £27,000 in 3 years' time.

  Prize C: Receive £32,000 in 5 years' time.

  The bank interest rate is estimated to be 10% pa for the foreseeable future.

- By finding the present value of each amount, establish which prize is worth the most to Charlie.

Answer to example 6

- Present value, $X = \dfrac{S}{(1 + r)^n}$
- Prize A: Present value = £20,000
- Prize B: Present value, $X = \dfrac{£27{,}000}{1.1^3}$

  = £20,285 (nearest £)
- Prize C: Present value, $X = \dfrac{£32{,}000}{1.1^5}$

  = £19,869 (nearest £)
- Prize B is worth most

Discounting the quick way

- We already know that present value, $X = \dfrac{S}{(1 + r)^n}$
- Or we could rewrite this to give $X = S \times \dfrac{1}{(1 + r)^n}$
- This is called the discount factor and can be found in our mathematical tables
- This gives an alternative and quick method of calculating the present value

Present value, X = S x Discount Factor (from tables)

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Present Value Table

Present value of 1 i.e. $(1 + r)^{-n}$

Where r = discount rate

n = number of periods until payment

Discount rate (r)

Periods (n)	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	
1	0·990	0·980	0·971	0·962	0·952	0·943	0·935	0·926	0·917	0·909	1
2	0·980	0·961	0·943	0·925	0·907	0·890	0·873	0·857	0·842	0·826	2
3	0·971	0·942	0·915	0·889	0·864	0·840	0·816	0·794	0·772	0·751	3
4	0·961	0·924	0·888	0·855	0·823	0·792	0·763	0·735	0·708	0·683	4
5	0·951	0·906	0·863	0·822	0·784	0·747	0·713	0·681	0·650	0·621	5
6	0·942	0·888	0·837	0·790	0·746	0·705	0·666	0·630	0·596	0·564	6
7	0·933	0·871	0·813	0·760	0·711	0·665	0·623	0·583	0·547	0·513	7
8	0·923	0·853	0·789	0·731	0·677	0·627	0·582	0·540	0·502	0·467	8
9	0·914	0·837	0·766	0·703	0·645	0·592	0·544	0·500	0·460	0·424	9
10	0·905	0·820	0·744	0·676	0·614	0·558	0·508	0·463	0·422	0·386	10
11	0·896	0·804	0·722	0·650	0·585	0·527	0·475	0·429	0·388	0·350	11
12	0·887	0·788	0·701	0·625	0·557	0·497	0·444	0·397	0·356	0·319	12
13	0·879	0·773	0·681	0·601	0·530	0·469	0·415	0·368	0·326	0·290	13
14	0·870	0·758	0·661	0·577	0·505	0·442	0·388	0·340	0·299	0·263	14
15	0·861	0·743	0·642	0·555	0·481	0·417	0·362	0·315	0·275	0·239	15

(n)	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	
1	0·901	0·893	0·885	0·877	0·870	0·862	0·855	0·847	0·840	0·833	1
2	0·812	0·797	0·783	0·769	0·756	0·743	0·731	0·718	0·706	0·694	2
3	0·731	0·712	0·693	0·675	0·658	0·641	0·624	0·609	0·593	0·579	3
4	0·659	0·636	0·613	0·592	0·572	0·552	0·534	0·516	0·499	0·482	4
5	0·593	0·567	0·543	0·519	0·497	0·476	0·456	0·437	0·419	0·402	5
6	0·535	0·507	0·480	0·456	0·432	0·410	0·390	0·370	0·352	0·335	6
7	0·482	0·452	0·425	0·400	0·376	0·354	0·333	0·314	0·296	0·279	7
8	0·434	0·404	0·376	0·351	0·327	0·305	0·285	0·266	0·249	0·233	8
9	0·391	0·361	0·333	0·308	0·284	0·263	0·243	0·225	0·209	0·194	9
10	0·352	0·322	0·295	0·270	0·247	0·227	0·208	0·191	0·176	0·162	10
11	0·317	0·287	0·261	0·237	0·215	0·195	0·178	0·162	0·148	0·135	11
12	0·286	0·257	0·231	0·208	0·187	0·168	0·152	0·137	0·124	0·112	12
13	0·258	0·229	0·204	0·182	0·163	0·145	0·130	0·116	0·104	0·093	13
14	0·232	0·205	0·181	0·160	0·141	0·125	0·111	0·099	0·088	0·078	14
15	0·209	0·183	0·160	0·140	0·123	0·108	0·095	0·084	0·074	0·065	15

Illustration of discounting (part 2)

- Use the present value table to find the present value of £25,000 receivable in 6 years' time, if the interest rate is 10% pa.

Solution

- Present value, X = S x Discount factor $_{\text{6 years at 10\%}}$

  = £25,000 x 0.564

  = <u>£14,100</u>

- Note: in part 1 of the illustration our answer was £14,112. This is a rounding difference only

Example 7

- Use the present value tables to rework the answer for example 6

(5) Annuities

- Questions may require us to calculate the present value of a constant amount

- An annuity is a constant amount paid or received for a number of periods

Illustration of annuities

- Suppose I expect to receive £1,000 per annum for 3 years, starting in one year's time, and want to calculate the present value using a discount rate of 5%.

Answer

There are actually three methods available:

Method 1

- One approach would be to discount each cash flow separately and sum the results:
- Present value, X = S x Discount factor

Present value for year 1 amount = £1,000 x 0.952 = £952
Present value for year 2 amount = £1,000 x 0.907 = £907
Present value for year 3 amount = £1,000 x 0.864 = £864
2.723 £2,723

- The present value of £2,723 is correct but this is a time consuming method, particularly if the annuity continues for a long period

Method 2

- This is a quicker method than method 1. Rather than discounting each cash flow individually we can discount the annuity using a cumulative present value.
- This is the sum of all of the individual discount factors and can be found from the tables.

Present value of an annuity = annuity x cumulative present value factor

- Using tables this = £1,000 x 2.723 (as seen in method 1)

  = £2,723 (method 1 gave the same answer)

Method 3

- The cumulative present value factor may not be available from the tables. They are only available for whole numbers from 1% to 20%
- The calculation is the same as in method 2 but we will need to calculate the cumulative present value factor.

Present value of an annuity = annuity x cumulative present value factor

- The cumulative present value factor is calculated using the formula:

$$\frac{1}{r}\left[1-\frac{1}{(1+r)^n}\right]$$ (given)

PV = £1,000 x $\frac{1}{0.05}\left[1-\frac{1}{(1+0.05)^3}\right]$

= £2,723 (as before)

Annuity Table

Present value of an annuity of 1 i.e. $\frac{1-(1+r)^{-n}}{r}$

Where r = discount rate
n = number of periods

Discount rate (r)

Periods (n)	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	
1	0·990	0·980	0·971	0·962	0·952	0·943	0·935	0·926	0·917	0·909	1
2	1·970	1·942	1·913	1·886	1·859	1·833	1·808	1·783	1·759	1·736	2
3	2·941	2·884	2·829	2·775	2·723	2·673	2·624	2·577	2·531	2·487	3
4	3·902	3·808	3·717	3·630	3·546	3·465	3·387	3·312	3·240	3·170	4
5	4·853	4·713	4·580	4·452	4·329	4·212	4·100	3·993	3·890	3·791	5
6	5·795	5·601	5·417	5·242	5·076	4·917	4·767	4·623	4·486	4·355	6
7	6·728	6·472	6·230	6·002	5·786	5·582	5·389	5·206	5·033	4·868	7
8	7·652	7·325	7·020	6·733	6·463	6·210	5·971	5·747	5·535	5·335	8
9	8·566	8·162	7·786	7·435	7·108	6·802	6·515	6·247	5·995	5·759	9
10	9·471	8·983	8·530	8·111	7·722	7·360	7·024	6·710	6·418	6·145	10
11	10·368	9·787	9·253	8·760	8·306	7·887	7·499	7·139	6·805	6·495	11
12	11·255	10·575	9·954	9·385	8·863	8·384	7·943	7·536	7·161	6·814	12
13	12·134	11·348	10·635	9·986	9·394	8·853	8·358	7·904	7·487	7·103	13
14	13·004	12·106	11·296	10·563	9·899	9·295	8·745	8·244	7·786	7·367	14
15	13·865	12·849	11·938	11·118	10·380	9·712	9·108	8·559	8·061	7·606	15

(n)	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	
1	0·901	0·893	0·885	0·877	0·870	0·862	0·855	0·847	0·840	0·833	1
2	1·713	1·690	1·668	1·647	1·626	1·605	1·585	1·566	1·547	1·528	2
3	2·444	2·402	2·361	2·322	2·283	2·246	2·210	2·174	2·140	2·106	3
4	3·102	3·037	2·974	2·914	2·855	2·798	2·743	2·690	2·639	2·589	4
5	3·696	3·605	3·517	3·433	3·352	3·274	3·199	3·127	3·058	2·991	5
6	4·231	4·111	3·998	3·889	3·784	3·685	3·589	3·498	3·410	3·326	6
7	4·712	4·564	4·423	4·288	4·160	4·039	3·922	3·812	3·706	3·605	7
8	5·146	4·968	4·799	4·639	4·487	4·344	4·207	4·078	3·954	3·837	8
9	5·537	5·328	5·132	4·946	4·772	4·607	4·451	4·303	4·163	4·031	9
10	5·889	5·650	5·426	5·216	5·019	4·833	4·659	4·494	4·339	4·192	10
11	6·207	5·938	5·687	5·453	5·234	5·029	4·836	4·656	4·486	4·327	11
12	6·492	6·194	5·918	5·660	5·421	5·197	4·988	4·793	4·611	4·439	12
13	6·750	6·424	6·122	5·842	5·583	5·342	5·118	4·910	4·715	4·533	13
14	6·982	6·628	6·302	6·002	5·724	5·468	5·229	5·008	4·802	4·611	14
15	7·191	6·811	6·462	6·142	5·847	5·575	5·324	5·092	4·876	4·675	15

Example 8

- A new machine will generate an additional £3,333 per annum for the next four years. The first cash flow starts in one year and the discount rate is 5%

(a) Calculate the present value of the annuity using the annuity formula

(b) Recalculate the present value of the annuity using the tables

(Answers should be rounded to the nearest £)

Annuities in advance and delayed annuities

- The calculation seen will only work for ‘normal’ annuities. This is one that begins in one year

- The calculation must be altered slightly if the annuity is in advance, i.e. begins immediately or is delayed, i.e. begins after year 1

Illustration of annuities in advance and delayed annuities

- Using your the information in example 8, calculate the present value of the annuity using the tables if

(a) The annuity begins immediately

(b) The annuity begins in three years time

(Answers should be rounded to the nearest £)

Answer

(a) The first annuity does not need discounting since it is received immediately. The following three annuities are received in years 1 -3 and will be discounted in the normal way using the 3 year cumulative present value factor.

PV of the annuity = £3,333 + (£3,333 x 2.723)

= £12,409

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(b) The annuities are received in years 3 -6. When we discount we need to use the cumulative present value factor for year 6 less the cumulative present value factor for year 2. This will give us the correct cumulative present value factor for years 3 -6.

PV of the annuity = £3,333 x (5.076 – 1.859)

= £10,722

(6) Perpetuity

- A perpetuity is an annuity that continues forever.

Present value of a perpetuity =
perpetuity x 1/r

r = cost of capital/ discount rate

Illustration of a perpetuity

- Jo is looking to purchase a perpetuity that guarantees a payment of £10,000 per annum. What is a fair price for the perpetuity, assuming a discount rate of 3% per annum? (Round the answer to the nearest £)

Answer

- Present value of the perpetuity = £10,000 x 1/0.03

= £333,333

(7) Net present value (NPV)

- The NPV method is used to appraise investments and involves discounting.

NPV = present value of all the cash inflows minus the present value of all the cash outflows.

- If the NPV is positive we accept the project.

Illustration of NPV (part 1)

- A company is considering investing £100,000 in a project, which is forecast to yield the following net cash flows:

Year	1	2	3	4	5
Net cash flow (£000)	40	35	32	25	19

- Calculate the net present value of this project if the firm has a cost of capital of 10%

Answer

Time	Cash Flow £000	Discount Factor 10%	Present Value £000
0	(100)	1	(100)
1	40	0.909	36.36
2	35	0.826	28.91
3	32	0.751	24.032
4	25	0.683	17.075
5	19	0.621	11.799
			18.176

- Project has a positive NPV of £18,176: accept

Illustration of NPV (part 2)

- A new project requires £100,000 investment and is expected to generate cash inflows of £35,000 per annum for 5 years. Evaluate the project at a discount rate of 15%.

Answer

- Each of the cash inflows in years 1 -5 are an annuity. Therefore, the method for discounting annuities should be used

Time	Cash Flow (£)	Discount Factor 15%	Present Value (£)
Year 0	(100,000)	1	(100,000)
Year 1 -5	35,000	3.352	117,320
			17,320

- NPV is positive: accept the project

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Example 9

- A machine costs £45,000 and has a residual value of £5,000. It will generate savings of £9,000 per annum for the next 8 years. The company's cost of capital is 7%. Should the machine be purchased?

Answer to example 9

- Exam tip: Remember to include any residual value in the cash flow for the final year

Time	Cash Flow (£)	Discount Factor 7%	Present Value (£)
0	(45,000)	1	(45,000)
1-7	9,000	5.389	48,501
8	14,000	0.582	<u>8,148</u>
			<u>11,649</u>

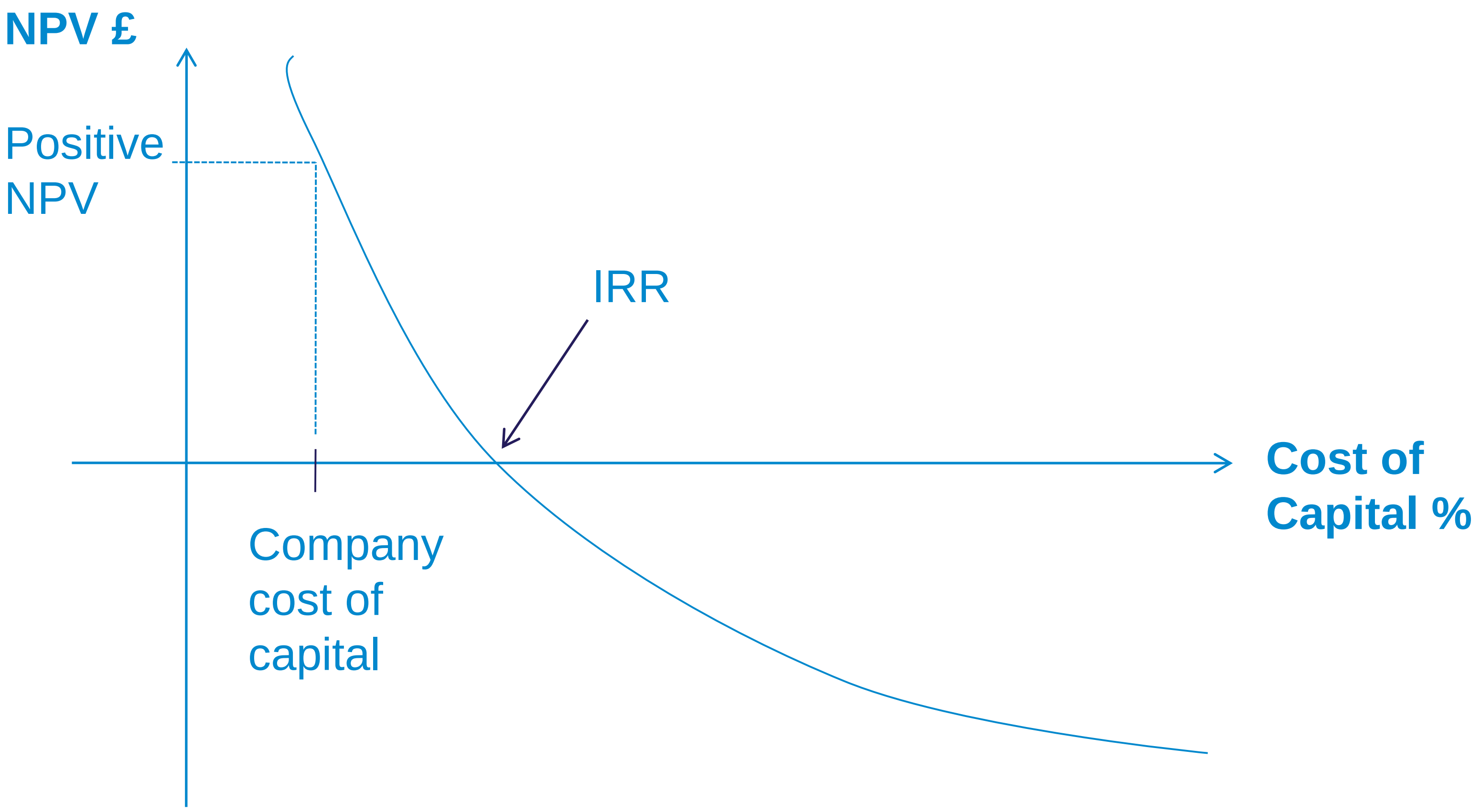
- The project has a NPV of £11,649: accept

(8) Internal rate of return, IRR

- IRR is another method of appraising investments and involves discounting

The IRR is the discount rate at which NPV is zero

- Accept the project if the IRR is more than the company's cost of capital


NPV £
Positive
NPV
IRR
Cost of
Capital %
Company
cost of
capital

Estimate of IRR

- The process on the previous slide is too time consuming. An estimated IRR is calculated using a three step approach:
- Step 1: Take a small discount rate r_1 and calculate the NPV (NPV_1)
- Step 2: Take another discount rate r_2 and calculate the NPV (NPV_2)
- Step 3: Use the formula to calculate the IRR

$$\mathrm{IRR} = r_1 + \left[\frac{\mathrm{NPV}_1}{\mathrm{NPV}_1 - \mathrm{NPV}_2}\left(r_2 - r_1\right)\right]$$ (learn)

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Illustration of IRR

- A project involves investing £140,000, and will produce the following year-end cash flows:

Year	1	2	3	4
Net cash flow (£000)	60	50	45	30

Discount the project at 10% and at 15%, then calculate the internal rate of return of the project

Answer

Year	Cash Flow (£)	Discount Factor 15%	Present Value (£)	Discount Factor 10%	Present Value (£)
0	(140,000)	1	(140,000)	1	(140,000)
1	60,000	0.870	52,200	0.909	54,540
2	50,000	0.756	37,800	0.826	41,300
3	45,000	0.658	29,610	0.751	33,795
4	30,000	0.572	17,160	0.683	20,490
			(3,230)		**10,125**

Step 3: Calculate the IRR

$$\mathrm{IRR} = r_1 + \left[\frac{\mathrm{NPV}_1}{\mathrm{NPV}_1 - \mathrm{NPV}_2} (r_2 - r_1) \right]$$

$$\mathrm{IRR} = 10 + \left[\frac{10{,}125}{(10{,}125 - (-3{,}230))} \times (15 - 10) \right]$$

$$= \underline{13.79\%}$$

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Example 10

- Regulus is considering investing in a project and has calculated the following NPVs:

  At a discount rate of 5% NPV = +£50,000

  At a discount rate of 15% NPV = -£27,000

  Calculate the IRR of the project and advise Regulus if his normal discount rate is 10%